P425/1
PURE
MATHEMATICS
Paper 1
23rd February, 2024
3 hours

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

Attempt **ALL** *the eight questions in* Section **A** *and any* **five** *from* Section **B.**

Begin every answer on a fresh page.

Any additional questions answered will **not** *be marked.*

Mathematical tables and squared paper shall be provided

Silent, non – programmable calculators may be used.

State the degree of accuracy at the end of each answer attempted using a calculator or table and indicate **cal** *for calculator or* **tab** *for mathematical table.*

SECTION A: 40 MARKS

Attempt **all** *questions in this* Section.

1. Solve the equation $1 - 2\sin\theta - 4\cos 2\theta = 0$ for values of 0 between 0^0 and 360^0. *(05 marks)*

2. If $(x-2)^2$ is a factor of $x^3 + ax^2 + bx + 12$. Find the values of a and b. *(05 marks)*

3. Find the distance from the centre of the circle $x^2 + y^2 - 4x + 2y - 2 = 0$ to the line $3x - 4y + 5 = 0$. *(05 marks)*

4. Differentiate $\sin^2 x$ from first principles. *(05 marks)*

5. Find the possible values of k if the quadratic equation $2kx^2 - 8x + 1 = 2k(x-2)$ has equal roots. *(05 marks)*

6. Use Maclaurin's theorem to find the first 3 non zero terms of the expansion $\ln(1-2x)$. *(05 marks)*

7. Find the coordinates of the point of intersection of the lines

$r = \begin{pmatrix} 9 \\ -3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$ and $r = \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ *(05 marks)*

8. Find $\int \frac{2x^2}{\cos^2(x^3+2)}\,dx$ *(05 marks)*

SECTION B

9. (a) Given that $z_1 = 2$ and $z_2 = 3 + 2\boldsymbol{i}$. find the argument of $\left(\frac{z_1 + z_2}{z_2 - z_1}\right)$ *(06 marks)*

(b) Given that $z = -3$ is a root of the equation $z^3 - 5z^2 + az + 60 = 0$. Find the value of a and hence find other roots. *(06 marks)*

10. (a) Find the mean value of the function $y = \sqrt{x}$ from $x = 0$ to $x = 4$. *(05 marks)*

(b) Given that $y = \ln \sin t$ and $x = 1 - \cos t$. find in $\frac{d^2y}{dx^2}$ terms of t. *(07 marks)*

11. (a) Show that $\frac{\cos(2x + 60^0) - \cos(2x - 60^0)}{\cos(x + 30^0) - \cos(x - 30^0)} = 2\sqrt{3}\cos x$ *(05 marks)*

(b) Given that $\tan A = \frac{4}{3}$ and $\tan(A + B)\ \frac{63}{16}$ Find the value of $\sin(A - B)$ without using tables or calculators if A and B are acute angles. *(07 marks)*

12. (a) Expand $(1 + 2x)^{\frac{1}{2}}$ in ascending powers of x up to the term in x^3. By putting $x = \frac{1}{25}$ find an approximate value of $\sqrt{3}$ to 3 decimal places. *(06 marks)*

(b) Given that $(1 + 3x)^n = 1 + 15x + 6mx^2 + \ldots$. Find the values of constants m and n. *(06 marks)*

13. (a) If vectors $3\boldsymbol{i} + a\boldsymbol{j} + 2\boldsymbol{k}$ and $-6\boldsymbol{i} + a\boldsymbol{j} + \boldsymbol{k}$ are perpendicular, find the value of a given that a < 0. *(05 marks)*

(b) Determine the coordinates of the point of intersection of the line $\frac{x+1}{3} = \frac{y-3}{3} = Z - 8$ and the plane $x - 2y - 3Z = -37$. *(07 marks)*

14. The gradient of a curve at the point p (x, y) is $\frac{x^2 + 1}{y}$ The point A (3, 12) lies on the curve. Find:

(i) equation of the normal to the curve at A. *(05 marks)*

(ii) equation of the curve and the point where it cuts the y – axis. *(07 marks)*

15. (a) Solve for x if $\tan^{-1}x + \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{8}\right) = \frac{\pi}{4}$. *(06 marks)*

(b) Show that if A, B and C are angles of a triangle then
$a\cos 2B + 2b\cos A\cos B = c\cos B - b\cos C$. *(06 marks)*

16. The sum of the first terms of an AP and GP is 57. The sum of the second terms of the same AP and G.P is 94. The sum of the third terms of the AP and G.P is 171. If the common ratio of the G.P is 2. Find the first term of the progression and the common difference of the AP and the sum of the first 20 terms of the A.P. *(12 marks)*

END